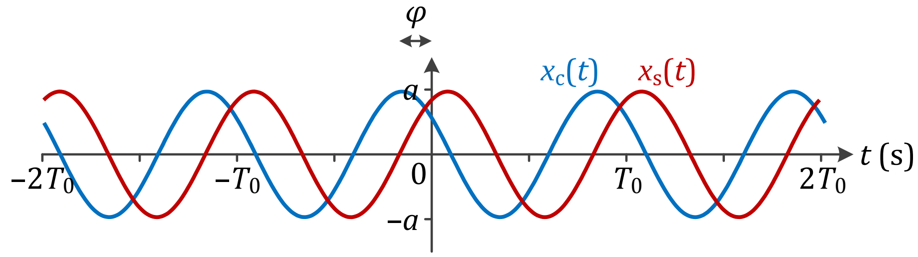
Sine Wave

Time domain

$x_c(t) = a\cos(2\pi f_0 t + \varphi) = a\cos(\omega_0 t + \varphi)$

$x_s(t) = a\sin(2\pi f_0 t + \varphi) = a\sin(\omega_0 t + \varphi)$

$x_e(t) = x_c(t) + \mathrm{j}\, x_s(t) = a\,\mathrm{e}^{\mathrm{j}(2\pi f_0 t + \varphi)} = a\,\mathrm{e}^{\mathrm{j}(\omega_0 t + \varphi)}$



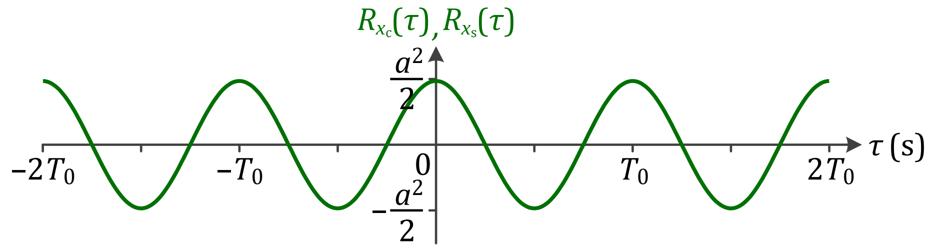
a : amplitude
f_0 : frequency (Hz)
ω_0: angular frequency (rad/s)
T_0 : period (s)
φ : initial phase (rad)
t : time (s)

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$

Autocorrelation

$$R_x(\tau) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)x^*(t-\tau)\mathrm{d}t = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t+\tau)x^*(t)\mathrm{d}t\,,\ \forall\, t_0$$

τ : time shift (s)



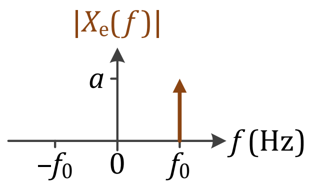
$R_{x_e}(\tau) = a^2\mathrm{e}^{-\mathrm{j}2\pi f_0 \tau}$

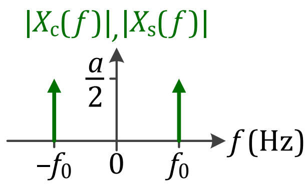
$\forall\, f_0 \neq 0$:

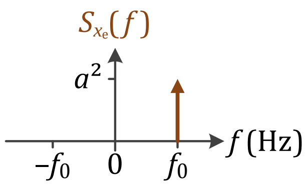
$R_{x_c}(\tau) = R_{x_s}(\tau) = \frac{a^2}{2}\cos(2\pi f_0 \tau)$

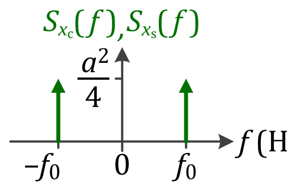
Frequency domain

Fourier transform	Power spectral density
$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi f t}\mathrm{d}t$	$S_x(f) = \mathcal{F}\{R_x(\tau)\}$
$X_e(f) = \mathcal{F}\{x_e(t)\} = a\,\mathrm{e}^{\mathrm{j}\varphi}\delta(f-f_0)$	$S_{x_e}(f) = a^2\,\delta(f-f_0)$
$X_c(f) = \mathcal{F}\{x_c(t)\} = \frac{a}{2}\left[\mathrm{e}^{\mathrm{j}\varphi}\delta(f+f_0) + \mathrm{e}^{-\mathrm{j}\varphi}\delta(f-f_0)\right]$	$\forall\, f_0 \neq 0$:
$X_s(f) = \mathcal{F}\{x_s(t)\} = \mathrm{j}\frac{a}{2}\left[\mathrm{e}^{-\mathrm{j}\varphi}\delta(f+f_0) - \mathrm{e}^{\mathrm{j}\varphi}\delta(f-f_0)\right]$	$S_{x_c}(f) = S_{x_s}(f) = \frac{a^2}{4}\left[\delta(f+f_0) + \delta(f-f_0)\right]$



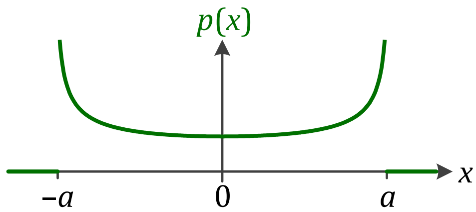






Distribution

$$p(x) = \begin{cases} \dfrac{1}{\pi\sqrt{a^2 - x^2}} = \dfrac{1}{\pi a\sqrt{(1-(x/a)^2}}, & |x| < a \\ 0, & |x| \geq a \end{cases},\quad \forall\, f_0 \neq 0$$



Power

$P_{x_e} = a^2$

$\forall\, f_0 \neq 0$:

$P_{x_c} = P_{x_s} = \frac{a^2}{2}$

From time domain: Mean of the square	From autocorrelation: Value for no delay	From frequency domain: Integral of the PSD	From distribution: Second moment
$P_x = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} \lvert x(t)\rvert^2\mathrm{d}t$	$P_x = R_x(0)$	$P_x = \int_{-\infty}^{\infty} S_x(f)\mathrm{d}f$	$P_x = \int_{-\infty}^{\infty} x^2 p(x)\mathrm{d}x$

Dirac delta

Properties

$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \text{unbounded}, & t = 0 \end{cases},\quad \int_{-\infty}^{\infty}\delta(t)\mathrm{d}t = 1$

$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0),\quad \int_{-\infty}^{\infty} x(t)\delta(t-t_0)\mathrm{d}t = x(t_0)$

Some equivalences

$\delta(t) = \lim_{T\to 0}\begin{cases} 1/T, & \lvert t\rvert \leq T/2 \\ 0, & \text{elsewhere} \end{cases}$

$\delta(t) = \mathcal{F}\{1\} = \int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{j}2\pi f t}\mathrm{d}t = \lim_{T\to\infty} T\,\mathrm{sinc}(\pi f T)$

