Mathematical Series

	Series	Alternating series
Integers	$\sum_{n=1}^{\infty}\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots=\infty$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots=\ln(2)\approx 0.693$
Odd integers	$\sum_{n=1}^{\infty}\frac{1}{2n-1}=\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots=\infty$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots=\frac{\pi}{4}\approx 0.785$
Even integers	$\sum_{n=1}^{\infty}\frac{1}{2n}=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\cdots=\infty$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n}=\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}-\cdots=\frac{\ln(2)}{2}\approx 0.347$
Prime numbers	$\sum_{n=1}^{\infty}\frac{1}{p_n}=\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{11}+\cdots=\infty$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{p_n}=\frac{1}{2}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{11}-\cdots\approx -0.270$
Fibonacci numbers	$\sum_{n=1}^{\infty}\frac{1}{F_n}=\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{8}+\frac{1}{13}+\cdots\approx 3.360$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{F_n}=\frac{1}{1}-\frac{1}{1}+\frac{1}{2}-\frac{1}{3}+\frac{1}{5}-\frac{1}{8}+\frac{1}{13}-\cdots\approx 0.289$
Factorials	$\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\cdots=\mathrm{e}\approx 2.718$	$\sum_{n=0}^{\infty}\frac{(-1)^{n}}{n!}=\frac{1}{1}-\frac{1}{1}+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}+\cdots=\frac{1}{\mathrm{e}}\approx 0.368$
Palindromic numbers	$\sum_{n\text{ palindrome}}^{\infty}\frac{1}{n}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{9}+\frac{1}{11}+\frac{1}{22}+\cdots\frac{1}{99}+\frac{1}{101}+\frac{1}{111}+\frac{1}{121}+\cdots+\frac{1}{191}+\frac{1}{202}+\cdots\approx 3.370$	
Integers excluding a given digit	Excluding 0: $K_0=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{9}+\frac{1}{11}+\frac{1}{12}+\cdots+\frac{1}{19}+\frac{1}{21}+\frac{1}{22}+\cdots+\frac{1}{29}+\frac{1}{31}+\cdots\approx 23.10$; $K_1\approx 16.18$; $K_2\approx 19.26$; $K_3\approx 20.57$; $K_4\approx 21.33$; $K_5\approx 21.83$; $K_6\approx 22.21$; $K_7\approx 22.49$; $K_8\approx 22.73$; $K_9\approx 22.92$	
Triangular numbers	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)/2}=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\cdots=2$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+1)/2}=\frac{1}{1}-\frac{1}{3}+\frac{1}{6}-\frac{1}{10}+\frac{1}{15}-\cdots=4\ln(2)-2$
Tetrahedral numbers	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)/6}=\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+\frac{1}{35}+\cdots=\frac{3}{2}$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+1)(n+2)/6}=\frac{1}{1}-\frac{1}{4}+\frac{1}{10}-\cdots=12\ln(2)-\frac{15}{2}$
Pentagonal numbers	$\sum_{n=1}^{\infty}\frac{1}{n(3n-1)/2}=\frac{1}{1}+\frac{1}{5}+\frac{1}{12}+\frac{1}{22}+\cdots=3\ln(3)-\frac{\pi}{\sqrt{3}}$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(3n-1)/2}=\frac{1}{1}-\frac{1}{5}+\frac{1}{12}-\frac{1}{22}+\cdots=\frac{2\pi}{\sqrt{3}}-4\ln(2)$
Hexagonal numbers	$\sum_{n=1}^{\infty}\frac{1}{n(2n-1)}=\frac{1}{1}+\frac{1}{6}+\frac{1}{15}+\frac{1}{28}+\cdots=2\ln(2)$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(2n-1)}=\frac{1}{1}-\frac{1}{6}+\frac{1}{15}-\frac{1}{28}+\cdots=\frac{\pi}{2}-\ln(2)$
Product of two consecutive numbers	$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots=1$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+1)}=\frac{1}{1\times 2}-\frac{1}{2\times 3}+\frac{1}{3\times 4}-\cdots=2\ln(2)-1$
	$\sum_{n=1}^{\infty}\frac{1}{(2n-1)2n}=\frac{1}{1\times 2}+\frac{1}{3\times 4}+\frac{1}{5\times 6}+\cdots=\ln(2)$	$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)2n}=\frac{1}{1\times 2}-\frac{1}{3\times 4}+\frac{1}{5\times 6}-\cdots=\frac{\pi}{4}-\frac{\ln(2)}{2}$

	Series	Alternate series
Product of three consecutive numbers	$\sum_{n=1}^{\infty} \frac{1}{(2n-1)2n(2n+1)} = \frac{1}{1\times 2\times 3} + \frac{1}{3\times 4\times 5} + \cdots = \ln(2) - \frac{1}{2}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)2n(2n+1)} = \frac{1}{1\times 2\times 3} - \frac{1}{3\times 4\times 5} + \cdots = \frac{1-\ln(2)}{2}$
	$\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n-1)3n} = \frac{1}{1\times 2\times 3} + \frac{1}{4\times 5\times 6} + \cdots = \frac{\pi}{4\sqrt{3}} - \frac{\ln(3)}{4}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(3n-2)(3n-1)3n} = \frac{1}{1\times 2\times 3} - \frac{1}{4\times 5\times 6} + \cdots = \frac{2\ln(2)}{3} - \frac{\pi}{6\sqrt{3}}$
	$\sum_{n=1}^{\infty} \frac{1}{2n(2n+1)(2n+2)} = \frac{1}{2\times 3\times 4} + \frac{1}{4\times 5\times 6} + \cdots = \frac{3}{4} - \ln(2)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n(2n+1)(2n+2)} = \frac{1}{2\times 3\times 4} - \frac{1}{4\times 5\times 6} + \cdots = \frac{\pi-3}{4}$
	$\sum_{n=1}^{\infty} \frac{1}{(3n-1)3n(3n+1)} = \frac{1}{2\times 3\times 4} + \frac{1}{5\times 6\times 7} + \cdots = \frac{\ln(3)-1}{2}$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(3n-1)3n(3n+1)} = \frac{1}{2\times 3\times 4} - \frac{1}{5\times 6\times 7} + \cdots = \frac{1}{2} - \frac{2\ln(2)}{3}$
Squares	$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6} \approx 1.645$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12} \approx 0.822$
Odd squares	$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8} \approx 1.234$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots \approx 0.916$
Even squares	$\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{24} \approx 0.411$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)^2} = \frac{1}{2^2} - \frac{1}{4^2} + \frac{1}{6^2} - \cdots = \frac{\pi^2}{48} \approx 0.206$
Cubes	$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots \approx 1.202$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} = \frac{1}{1^3} - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \cdots = \frac{3\zeta(3)}{4} \approx 0.902$
Odd cubes	$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} + \cdots = \frac{7\zeta(3)}{8} \approx 1.052$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{32} \approx 0.969$
Even cubes	$\sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{2^3} + \frac{1}{4^3} + \frac{1}{6^3} + \frac{1}{8^3} + \cdots = \frac{\zeta(3)}{8} \approx 0.150$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)^3} = \frac{1}{2^3} - \frac{1}{4^3} + \frac{1}{6^3} - \frac{1}{8^3} + \cdots = \frac{3\zeta(3)}{32} \approx 0.113$
Tetration	$\sum_{n=1}^{\infty} \frac{1}{n^n} = \frac{1}{1^1} + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \cdots \approx 1.291$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^n} = \frac{1}{1^1} - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \cdots \approx 0.783$
Powers of two	$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \cdots = \frac{1}{3}$
Exponentials	$\sum_{n=1}^{\infty} \frac{1}{k^n} = \frac{1}{k} + \frac{1}{k^2} + \frac{1}{k^3} + \frac{1}{k^4} + \cdots = \frac{1}{k-1}$, if $\lvert k\rvert > 1$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{k^n} = \frac{1}{k} - \frac{1}{k^2} + \frac{1}{k^3} - \frac{1}{k^4} + \cdots = \frac{1}{k+1}$, if $\lvert k\rvert > 1$
Perfect powers	$\sum_{n \text{ perfect power with duplicates}}^{\infty} \frac{1}{n} = \sum_{n=2}^{\infty}\sum_{k=2}^{\infty} \frac{1}{n^k} = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots = 1$	
	$\sum_{n \text{ perfect power without duplicates}}^{\infty} \frac{1}{n-1} = \frac{1}{3} + \frac{1}{7} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{26} + \frac{1}{31} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63} + \frac{1}{81} + \frac{1}{99} + \cdots = 1$	